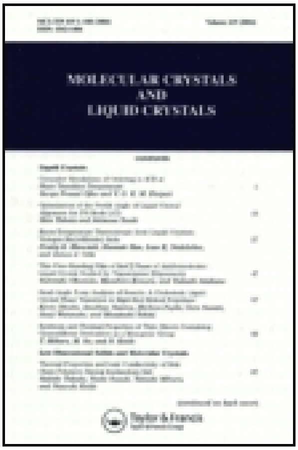


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# Shear Flow of a Ferronematic in a Magnetic Field

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# Shear Flow of a Ferronematic in a Magnetic Field

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*We examine the effect of shear flow on the orientational phase transitions induced by magnetic field in ferronematics. Continuum approach based on the generalized Leslie-Ericksen theory is used to describe the dynamics of a ferronematic. We consider the steady state shear flow of unbounded ferronematic with constant velocity gradient in the uniform magnetic field. Stationary solutions for the director and the magnetization are obtained as functions of the magnetic field strength for the different material parameters. Our results show that shear flow can leads to the shift of the field thresholds or to a "smoothing" of the transitions in a ferronematic.*

**Keywords:** ferronematic; phase transitions; shear flow

## INTRODUCTION

Ferronematics (FN) are suspensions of magnetic particles in nematic liquid crystals (NLC). The idea to dope the elongated ferromagnetic particles into the liquid crystal (LC) carrier to increase the magnetic susceptibility of LC structure was first proposed by Brochard and de Gennes [1]. The coupling of the magnetic particles with LC molecules causes two mechanisms of the magnetic field influence on the FN orientational structure: diamagnetic (the influence on the NLC-matrix) and ferromagnetic (the influence on the magnetic particles). These mechanisms lead to different orientational order of a FN in magnetic field. It is possible to discriminate phases with homeotropic, angular and planar types of the magnetic particles coupling with NLC-matrix [2]. Besides, the viscous anisotropic properties of NLC carrier

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allow to control FN structure. So, the steady state shear flow with constant velocity gradient aligns the molecules of a pure NLC at the some angle to the direction of flow, known as Leslie angle, which determines possible planar solutions for the NLC director field in the absence of external forces and boundary effects (the unbounded NLC) [3].

The goal of this work is to analyze the influence of shear flow on the orientational phases of a FN in the magnetic field, and define the boundaries of the phase transitions and their features. Generalized Ericksen-Leslie continuum theory is exploited to describe FN dynamics [1,4,5]. We consider the steady state shear flow of unbounded ferronematic in magnetic field. Stationary solutions for the planar director and magnetization fields are obtained as functions of the magnetic field strength, the coupling energy, and the velocity gradient of shear flow. We derive analytical expressions for the critical magnetic field strength, which determine transitions between orientational phases in weak shear flow, and asymptotic functions for the director and magnetization angles in strong magnetic fields.

## BASIC EQUATIONS

Continuum approach for description of ferroliquid crystals was first proposed in [1]. It is based on the generalized thermodynamic potential (free energy) of liquid crystal considering presence of small amount of single-domain ferroparticles in LC carrier. The authors [1] supposed a rigid coupling of LC with ferroparticles. It is not correct for real ferroliquid crystals, therefore Burylov and Raikher [5] offered a potential with soft surface coupling, which allow considering a director and magnetization fields as independent variables. The density of the bulk free energy of a ferronematic in magnetic field for soft director coupling can be written in the following form [2,5]

$$
\begin{aligned}
&F = F_1 + F_2 + F_3 + F_4 + F_5,\\
&F_1 = \frac{1}{2}[K_1(\nabla\cdot\boldsymbol{n})^2 + K_2(\boldsymbol{n}\cdot\nabla\times\boldsymbol{n})^2 + K_3(\boldsymbol{n}\times\nabla\times\boldsymbol{n})^2],\\
&F_2 = -M_s f\,\boldsymbol{m}\cdot\boldsymbol{H}, \quad F_3 = -\frac{1}{2}\chi_a(\boldsymbol{n}\cdot\boldsymbol{H})^2,\\
&F_4 = \frac{k_B T}{\nu} f\ln f, \quad F_5 = \frac{w}{d} f(\boldsymbol{n}\cdot\boldsymbol{m})^2. \qquad (1)
\end{aligned}
$$

Here $K_1$, $K_2$, $K_3$ are the splay, twist, and bend elastic modules of NLC (Frank constants), $\boldsymbol{n}$ is the director of a liquid crystal, $M_s$ is the saturation magnetization of the ferroparticle material, $f$ is the volume fraction of the particles in a suspension, $\boldsymbol{m}$ is the unit vector along magnetization of the suspension, $\chi_a$ is the anisotropy of a magnetic susceptibility (we assume that $\chi_a > 0$), $\nu$ is the volume of a ferroparticle, $k_B$ is the Boltzmann constant, $d$ is the transverse diameter of a ferroparticle, $T$ is the temperature, $w$ is the surface energy density of the NLC molecules coupling with magnetic particles (we assume that $w > 0$).

We consider FN with low volume fraction $f \ll 1$ of ferroparticles and wherefore we neglect the interparticle magnetic dipole-dipole interactions in a suspension. The first term $F_1$ represents the bulk free energy density of the director field elastic deformations of NLC (the Oseen-Frank potential). The second $F_2$ and the third $F_3$ contributions characterize the interactions of magnetic moments $\mu = M_s \nu$ of particles (the dipole mechanism of interaction) and diamagnetic NLC-matrix (the quadrupole mechanism of interaction) with an external magnetic field $\boldsymbol{H}$, respectively. The fourth term $F_4$ is the contribution of the mixing entropy of the ideal ferromagnetic particle solution. The last term $F_5$ determines the surface coupling energy of the magnetic particles with NLC-matrix (the coupling energy). We consider FN, which has homeotropic coupling conditions of NLC with magnetic particles ($\boldsymbol{\mu} \perp \boldsymbol{n}$). At $\boldsymbol{H} = 0$ these conditions determine the plane, which is normal to the director $\boldsymbol{n}$, and where the magnetic moments $\boldsymbol{\mu}$ of the particles are situated. If a FN is prepared from an isotropic suspension by cooling in the absence of external fields, the macroscopic magnetization $\boldsymbol{m}$ of the sample is absent. However, it is possible to take off the degeneration of the magnetic moments orientation in a suspension by during the passage of through the clearing point at the presence of the magnetic field. This FN possesses the uncompensated magnetization $\boldsymbol{m}$ in the absence of a magnetic field. Therefore, the description in terms of magnetization is valid for such FN even in zero magnetic field. For this case the coupling energy $F_5$ is minimized at $\boldsymbol{m} \perp \boldsymbol{n}$.

According to the Ericksen-Leslie continuum theory [1] the dynamic equations of a ferronematic, which present the balance of forces acting on the fluid and the incompressibility condition, can be written as

$$\rho \frac{d\boldsymbol{v}}{dt} = \nabla \cdot \boldsymbol{\sigma}, \tag{2}$$

$$\nabla \cdot \boldsymbol{v} = 0. \tag{3}$$

Here $\rho$, $\boldsymbol{v}$, and $\boldsymbol{\sigma}$ are the mass density, the velocity, and the total stress tensor of NLC carrier (the embedding of ferroparticles in NLC carrier produce negligible changes of density and stress tensor due to low volume fraction $f \ll 1$ of magnetic particles in ferronematic); $d/dt$ denotes the convective time derivative $\partial/\partial t + \boldsymbol{v}\cdot\nabla$.

The stress tensor $\boldsymbol{\sigma}$ in Eq. (2) is determined as a sum

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' + \boldsymbol{\sigma}^{(e)}, \tag{4}$$

where viscous stress tensor $\boldsymbol{\sigma}'$ has the form

$$\sigma'_{ki} = \alpha_1 n_k n_i n_l n_m A_{lm} + \alpha_2 n_k N_i + \alpha_3 n_i N_k + \alpha_4 A_{ki} + \alpha_5 n_k n_l A_{li} + \alpha_6 n_i n_l A_{lk},$$

here the summation over repeated indices is implied. The vector $\boldsymbol{N} = d\boldsymbol{n}/dt - \boldsymbol{\omega}\cdot\boldsymbol{n}$ represents the rate of change of the director $\boldsymbol{n}$ relative to the background liquid, $A_{ik} = (\nabla_k v_i + \nabla_i v_k)/2$ and $\omega_{ik} = (\nabla_k v_i - \nabla_i v_k)/2$ are symmetric and antisymmetric parts of the velocity gradients tensor, respectively. The six viscosity coefficients $\alpha_s$ are called Leslie coefficients.

The elastic part of stress tensor (4) known as Ericksen tensor $\boldsymbol{\sigma}^{(e)}$ is given by

$$\sigma^{(e)}_{ki} = -p\delta_{ki} - \frac{\partial F}{\partial(\nabla_k n_l)}\nabla_i n_l,$$

where $p$ is the pressure, $\delta_{ki}$ is the Kronecker symbol.

The dynamic equation for the director $\boldsymbol{n}$ [3] has the form

$$\boldsymbol{h}^{(n)} = \gamma_1 \boldsymbol{N} + \gamma_2 \boldsymbol{n}\cdot\boldsymbol{A}, \tag{5}$$

where the rotary viscosity coefficients of nematic are given by $\gamma_1 = \alpha_3 - \alpha_2$, $\gamma_2 = \alpha_2 + \alpha_3$. The coefficient $\gamma_1$ characterizes the viscous torque associated with an angular velocity of the director, while $\gamma_2$ gives the contribution to this torque due to a shear velocity in NLC.

The dynamic equation for the unit magnetization vector $\boldsymbol{m}$ [4] has the form

$$\boldsymbol{h}^{(m)} = \gamma_{1p}\boldsymbol{M} + \gamma_{2p}\boldsymbol{m}\cdot\boldsymbol{A}, \tag{6}$$

where $\gamma_{1p}$ and $\gamma_{2p}$ are the rotary viscosity coefficients of the ferroparticles. The vector $\boldsymbol{M} = d\boldsymbol{m}/dt - \boldsymbol{\omega}\cdot\boldsymbol{m}$ characterizes the rate of change of the magnetization vector $\boldsymbol{m}$ relative to the background NLC carrier.

The molecular fields $\boldsymbol{h}^{(n)}$ and $\boldsymbol{h}^{(m)}$ in Eqs. (5) and (6) are given by

$$h_i^{(n)} = -\frac{\partial F}{\partial n_i} + \nabla_k \frac{\partial F}{\partial(\nabla_k n_i)}, \qquad h_i^{(m)} = -\frac{\partial F}{\partial m_i} + \nabla_k \frac{\partial F}{\partial(\nabla_k m_i)}.$$

The director $\boldsymbol{n}$ and magnetization $\boldsymbol{m}$ are the unit vectors, therefore the variation of the free energy $F$ (see Eq. (1)) should be produced under auxiliary conditions $\boldsymbol{n}^2 = 1$ and $\boldsymbol{m}^2 = 1$ by Lagrangian coefficients method.

The diffusion equation of ferroparticles in NLC carrier (the particle-number conservation law) [4] can be written in the following form

$$\frac{\partial f}{\partial t} + \nabla \cdot (\boldsymbol{U} f) = 0. \tag{7}$$

Here $\boldsymbol{U} = -B_t \nabla(\nu F^{(m)}/f)$ is the velocity of ferroparticles relative to the NLC carrier, $B_t$ is the diffusion coefficient, $\nu$ is the volume of a particle, $F^{(m)} = F_2 + F_4 + F_5$ are the terms of the total free energy (1), which are caused by the embedding of ferroparticles into the NLC carrier.



## SHEAR FLOW OF A FERRONEMATIC IN A MAGNETIC FIELD

We consider shear flow of a ferronematic $\boldsymbol{v} = (u(z), 0, 0)$ with constant velocity gradient $A = du/dz$ directed along the $z$ axis (see Fig. 1). Let the uniform magnetic field $\boldsymbol{H} = (H_x, 0, H_z)$ be imposed to the ferronematic in the shear plane. Assuming that induced FN configuration is also planar, the director $\boldsymbol{n}$, i.e. the unit vector characterizing the preferred direction of the main axis of the anisometric NLC molecules, and the unit vector $\boldsymbol{m}$ oriented along the direction of the FN magnetization may be written as

$$\boldsymbol{n} = (\cos\varphi,\ 0,\ \sin\varphi), \quad \boldsymbol{m} = (-\sin\psi,\ 0,\ \cos\psi). \tag{8}$$

For the stationary uniform shear flow of FN with the uniform director and magnetization fields (8) the diffusion equation (7) is

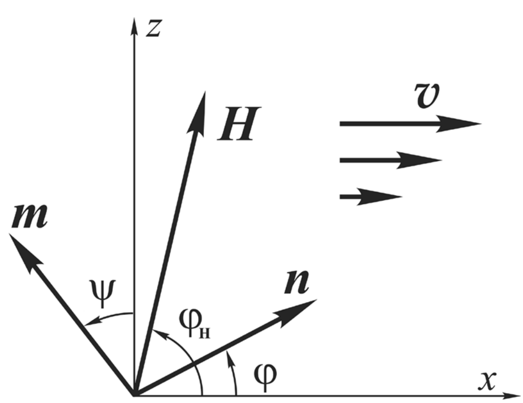


**FIGURE 1** Shear flow of a ferronematic in a magnetic field.

satisfied identically. The equations of FN motion (2) are significantly simplified:

$$\frac{du}{dz} = A, \qquad p = \frac{du}{dz}(\alpha_1 \sin^2 \varphi + \alpha_6) \sin \varphi \cos \varphi,$$

where $A = \text{const}$.

The projections of Eq. (5) are given by

$$\begin{aligned} &\chi_a H_x(\cos \varphi H_x + \sin \varphi H_z) + \frac{2wf}{d} \sin \psi \sin(\varphi - \psi) + \mu_1 \cos \varphi \\ &\quad = (\gamma_2 - \gamma_1)\frac{A}{2} \sin \varphi, \\ &\chi_a H_z(\cos \varphi H_x + \sin \varphi H_z) - \frac{2wf}{d} \cos \psi \sin(\varphi - \psi) + \mu_1 \sin \varphi \\ &\quad = (\gamma_2 + \gamma_1)\frac{A}{2} \cos \varphi, \end{aligned} \tag{9}$$

and the projections of Eq. (6) have the form

$$\begin{aligned} M_s f H_x &- \frac{2wf}{d} \cos \varphi \sin(\varphi - \psi) - \mu_2 \sin \psi \\ &= f(\gamma_{2p} - \gamma_{1p})\frac{A}{2} \cos \psi, \\ M_s f H_z &- \frac{2wf}{d} \sin \varphi \sin(\varphi - \psi) + \mu_2 \cos \psi \\ &= -f(\gamma_{2p} + \gamma_{1p})\frac{A}{2} \sin \psi. \end{aligned} \tag{10}$$

The elimination of Lagrangian coefficients $\mu_1$ and $\mu_2$ from Eqs. (9–10) and introduction of the polar coordinates for magnetic field strength $H_x = H \cos \varphi_H, H_z = H \sin \varphi_H$ (see Fig. 1) give the closed system, which determines the angles $\varphi$ and $\psi$ of the director $\boldsymbol{n}$ and magnetization $\boldsymbol{m}$, respectively:

$$\begin{aligned} &A(\gamma_1 + \gamma_2 \cos 2\varphi) + \chi_a H^2 \sin 2(\varphi - \varphi_H) + \frac{2wf}{d} \sin 2(\varphi - \psi) = 0, \\ &A(\gamma_{1p} - \gamma_{2p} \cos 2\psi) + 2M_s H \cos(\psi - \varphi_H) - \frac{2w}{d} \sin 2(\varphi - \psi) = 0. \end{aligned} \tag{11}$$

Let us introduce the parameters

$$\lambda = -\frac{\gamma_2}{\gamma_1}, \quad a_{1p} = \frac{\gamma_{1p}}{\gamma_1}, \quad a_{2p} = \frac{\gamma_{2p}}{\gamma_1}, \quad h = \frac{H}{H_0}, \quad \sigma = \frac{w\chi_a}{dM_s^2 f}, \quad \xi = \frac{A\gamma_1\chi_a}{M_s^2 f^2},$$

here $H_0 = M_s f/\chi_a$ is the unit of a magnetic field strength, at which the interaction energies of a nematic and magnetic particles with a magnetic field are of the same order. At $H \approx H_0$ occurs a replacement of the governing mechanism of the magnetic field influence on a FN from ferromagnetic (influence on the magnetic moments of the particles) to diamagnetic (influence on the NLC-matrix) or vice versa. Thus, at $H < H_0$ the ferromagnetic mechanism dominates on FN, and at $H > H_0$ – the diamagnetic one. The parameter $h$ is the dimensionless magnetic field strength; $\lambda$ is the reactive parameter (for real nematics [3] with rod-like molecules $\lambda > 0$); the coefficients $a_{1p}$ and $a_{2p}$ represent the ratio of rotary viscosity coefficients of ferroparticles and NLC; $\sigma$ determines the coupling energy of the magnetic particles with NLC-matrix, and $\xi$ is the dimensionless velocity gradient of shear flow. Choosing for the estimates [1,3,5] $\chi_a \sim 10^{-7}$ SGSE units, $\gamma_1, \gamma_2 \sim 0.1$ P, $\gamma_{1p}, \gamma_{2p} \sim 1\,\mathrm{P}$, $f \sim 10^{-6}$, $M_s \sim 10^2\,\mathrm{G}$, $w \sim 10^{-2}\,\mathrm{dyn/cm}$, $d \sim 10^{-5}\,\mathrm{cm}$, and the velocity gradient of shear flow $A \sim 0.1\,\mathrm{s}^{-1}$, we obtain $\lambda \sim 1$, $a_{1p}, a_{2p} \sim 10$, $\sigma \sim 0.01$, and $\xi \sim 0.1$.

Equations (11) can be expressed in the following dimensionless form

$$\begin{aligned} \xi(1 - \lambda\cos 2\varphi) + h^2 \sin 2(\varphi - \varphi_H) + 2\sigma \sin 2(\varphi - \psi) = 0 \\ f\xi(a_{1p} - a_{2p}\cos 2\psi) + 2h\cos(\psi - \varphi_H) - 2\sigma\sin 2(\varphi - \psi) = 0. \end{aligned} \tag{12}$$

From the estimations given above $f\xi \ll \sigma$, and so it is possible to neglect of shear flow influence on the ferroparticles orientation and Eq. (12) can be written as

$$\xi(1 - \lambda\cos 2\varphi) + h^2 \sin 2(\varphi - \varphi_H) + 2\sigma \sin 2(\varphi - \psi) = 0, \tag{13}$$

$$h\cos(\psi - \varphi_H) - \sigma\sin 2(\varphi - \psi) = 0. \tag{14}$$

The first term in Eq. (13) for the director angle characterizes the influence of shear flow on the NLC orientation, the second one describes the magnetic field effect on the NLC (the diamagnetic influence), and the third one defines the coupling of NLC with magnetic particles. The low volume fraction $f \ll 1$ of magnetic particles in the suspension allows to neglect of the flow influence on the particles orientation, and so the corresponding term is omitted in Eq. (14). The first term in Eq. (14) responds for the orientation of the FN magnetization vector in magnetic field (the ferromagnetic influence), besides, the direction of the magnetization in FN depends on the director orientation and the coupling energy of the magnetic particle with NLC-matrix, this effect is determined by the second term in Eq. (14).

Thus, Eqs. (13–14) determines the angles $\varphi$ and $\psi$ of the director $\boldsymbol{n}$ and magnetization $\boldsymbol{m}$ of a FN in shear flow as a functions of the strength $h$, the angle $\varphi_H$ of the magnetic field orientation, the reactive parameter $\lambda$, the velocity gradient $\xi$, and the coupling energy $\sigma$.

## MAGNETIC FIELD – INDUCED ORIENTATIONAL PHASES OF A FERRONEMATIC IN SHEAR FLOW

Without shear flow, in a ferronematic subjected to the uniform magnetic field the homeotropic, angular, and planar orientational phases (see Fig. 2) can be observed [2]. For low-field strength $(h \leq h_\perp = -\sigma + \sqrt{\sigma^2 + 2\sigma})$ the FN phase with homeotropic coupling between the director and the magnetization is stable, in which $\boldsymbol{n} \perp \boldsymbol{m} \parallel \boldsymbol{H}$. In this phase, the ordering of FN is ensured by ferromagnetic interactions with a field. When the magnetic field $h$ achieves the threshold value $h_\perp$, homeotropic phase is changed by the angular phase, in which the vectors $\boldsymbol{H}$, $\boldsymbol{m}$, and $\boldsymbol{n}$ are coplanar, but each two of these quantities encloses an angle which is different from 0 to $\pi/2$. The angles $\psi$ and $\varphi$, characterizing the orientations of the magnetization and the director, are functions of FN material parameters and magnetic field strength. In the angular phase, the orientations of the magnetic particles and the director are compromise between the ferromagnetic (at which $\boldsymbol{m} \parallel \boldsymbol{H}$) and diamagnetic $(\boldsymbol{n} \parallel \boldsymbol{H})$ ordering. With the field increasing the coupling conditions between the particles and the director are changed from homeotropic coupling $(\boldsymbol{n} \perp \boldsymbol{m})$ to planar one $(\boldsymbol{n} \parallel \boldsymbol{m})$, i.e., the director and magnetization rotate to a field direction due to $\chi_a > 0$ and finite coupling energy $w$. The angular phase is stable for $h_\perp < h < h_\parallel = \sigma + \sqrt{\sigma^2 + 2\sigma}$, and for $h \geq h_\parallel$ the saturation state is achieved, in which $\boldsymbol{n} \parallel \boldsymbol{m} \parallel \boldsymbol{H}$, and so the coupling between the particles and the director becomes planar. The transitions among the FN states have threshold character and can be classified as the second order transitions ($(\boldsymbol{n} \cdot \boldsymbol{m})^2$ plays a role of order parameter).

Let the FN be subjected to shear flow and magnetic field $\boldsymbol{H}$. The magnetic field is imposed in the shear plane at the angle

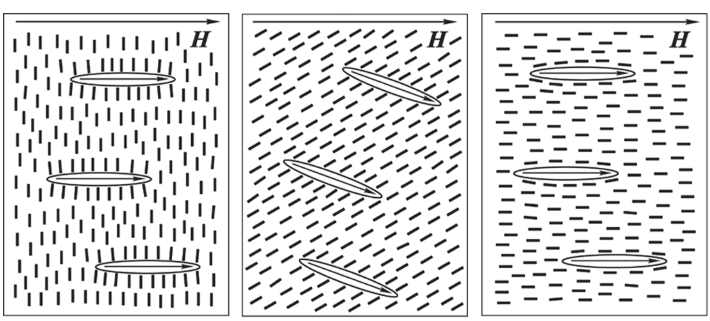


**FIGURE 2** Orientational phases of a ferronematic in a magnetic field.

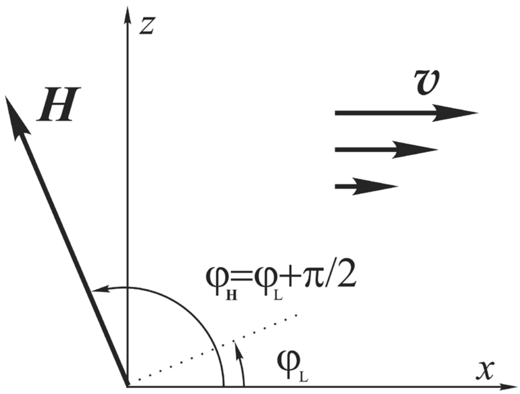


**FIGURE 3** The magnetic field $\boldsymbol{H}$ is imposed in shear plane at the angle $\varphi_H = \varphi_L + \pi/2$ orthogonally to the director $\boldsymbol{n}$ aligned by a shear flow only.

$\varphi_H = \varphi_L + \pi/2$ orthogonally to the director aligned by shear flow only (see Fig. 3). Here $\varphi_L = (1/2)\arccos(1/\lambda)$ is the flow alignment angle or Leslie angle, which determines the steady planar configurations of the NLC director in shear flow with constant velocity gradient [3]. The imposing of the magnetic field in such configuration allows to exist the homeotropic phase ($\boldsymbol{n} \perp \boldsymbol{m}$) in sheared FN. This orientational phase corresponds to the solutions $\varphi = \psi = \varphi_L$ of Eqs. (13–14). The director and the magnetization vectors are turned on the magnetic field direction at the increasing of its strength. This effect leads to the instability of the homeotropic phase, which is changed by the angular phase. We assume the solutions of Eqs. (13–14) in the following form: $\varphi = \varphi_L + \delta\varphi$, $\psi = \varphi_L + \delta\psi$. After the linearization on the small angles $\delta\varphi$ and $\delta\psi$ we obtain the equation for the threshold field $h_{\xi\perp}$ at which the angular orientational phase appears

$$h_{\xi\perp}^3 + 2\sigma h_{\xi\perp}^2 - (2\sigma + \beta)h_{\xi\perp} - 2\sigma\beta = 0, \tag{15}$$

here we introduce parameter $\beta = \xi\sqrt{\lambda^2 - 1}$. The results of the numerical solution of Eq. (5) are shown in Figure 4. In the absence of shear flow (Fig. 4, $\beta = 0$, solid line) expression (5) reduces to the relationship for the critical field $h_\perp$ in stationary case, which coincides with result obtained earlier [2]. As it is seen from Figure 4, the increase of $\beta$ (i.e. the increase of the velocity gradient or the reactive parameter) and $\sigma$ (the increase of coupling energy) results in the growth of the threshold magnetic field, besides, in the absence of the coupling ($\sigma = 0$) of NLC-matrix with magnetic particles (pure nematic) the threshold phenomenon takes place.

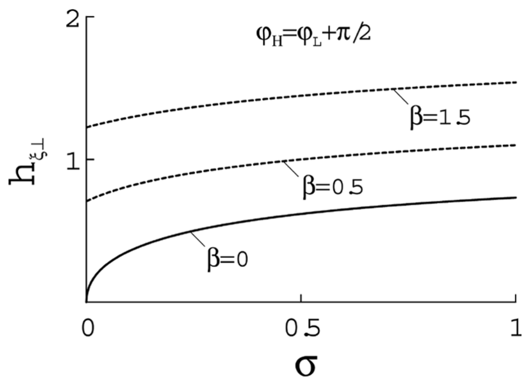


**FIGURE 4** The threshold magnetic field $h_{\xi\perp}$ as a function of the coupling energy $\sigma$ for the different values of the parameter $\beta = \xi\sqrt{\lambda^2 - 1}$.

Let's consider the case $\beta \ll 1$ corresponding to the weak velocity gradients ($\xi \ll 1$) or the small Leslie angle ($\lambda \approx 1$). The threshold value $h_{\xi\perp}$ is possible to present as the expansion in terms of the small parameter $\beta$ near the critical field $h_\perp$, which determines the threshold in the static case ($\beta = 0$). The first order expansion gives

$$h_{\xi\perp} = h_\perp \left[1 + \xi\sqrt{\lambda^2 - 1}/h_\perp^2(2 - h_\perp)\right]. \quad (16)$$

As it is seen from Eq. (16), the shear flow shift the stability threshold of the homeotropic phase in the region of the stronger fields then it was in the static case [2]. The additional term proportional to the velocity gradient is positive, because at the arbitrary values of the coupling energy $\sigma$ the critical field $h_\perp \leq 1$. Thus, the shear flow in considered magnetic field orientation ($\varphi_H = \varphi_L + \pi/2$) provides the stabilizing influence on the homeotropic phase ($\boldsymbol{n} \perp \boldsymbol{m}$) of the FN. At the equality of the magnitudes of rotary viscosity coefficients (which corresponds to the reactive parameter $\lambda = 1$) the threshold magnetic field in the presence of shear flow $h_{\xi\perp} \equiv h_\perp$, i.e., the critical strength coincides with the value corresponding to the case without flow ($\xi = 0$).

For the strong coupling ($\sigma \gg 1$) the threshold magnetic field (16) reduces to the expression $h_{\xi\perp} \approx 1 + \xi\sqrt{\lambda^2 - 1}$, which depends on the velocity gradient $\xi$ and reactive parameter $\lambda$ only. For the weak coupling ($\sigma \ll 1$) and the weak shear flow ($\beta \ll 1$) neglecting of $\sim \sigma\beta$ in Eq. (5) we obtain the critical field strength $h_{\xi\perp} \approx \sqrt{2\sigma + \xi\sqrt{\lambda^2 - 1}} - \sigma$.

Let's find the analytical expressions for the director angle $\varphi = \varphi(\xi, h, \sigma, \lambda)$ and magnetization orientational angle $\psi = \psi(\xi, h, \sigma, \lambda)$ in the strong magnetic fields ($h \gg 1$). In this case, the director $\boldsymbol{n}$ and the magnetization $\boldsymbol{m}$ tend to align in the direction of the magnetic field $\boldsymbol{H}$ ($\varphi \to \varphi_L + \pi/2, \psi \to \varphi_L$). As it is seen from Eqs. (13–14), in the limit of $h \gg 1$ the following asymptotic behavior yields

$$\varphi = \varphi_L + \frac{\pi}{2} - \frac{\xi}{h^2}, \quad \psi = \varphi_L + \frac{2\sigma\xi}{h^3}. \tag{17}$$

The results of Eqs. (13–14) numerical simulations are shown in Figures 5 and 6 for the different magnetic field strengths $h$ and velocity gradients $\xi$. As it is seen from these figures, the angular phase, in which the magnetization $\boldsymbol{m}$ make sharp angle with the director $\boldsymbol{n}$, arises at $h \geq h_{\xi\perp}$. Further increasing of the magnetic field strength leads to rotation of the director and the magnetization in the direction of a field, i.e., FN tends to the planar phase ($\boldsymbol{n} \parallel \boldsymbol{m}$). In the static case ($\xi = 0$) the transition to this state is realized by a threshold manner at $h \geq h_{\parallel}$ [2]. In the presence of shear flow the FN asymptotically comes to the planar phase in the limit of the infinitely large magnetic fields $h \to \infty$ (see Eq. (17)). The increasing of the velocity gradient $\xi$ leads to the stabilization of the homeotropic phase, shifting the threshold in the region of the stronger magnetic fields then it was in the static case. It agrees with expression (16) for the threshold field $h_{\xi\perp}$, obtained above in the limit of small velocity gradient $\xi$.

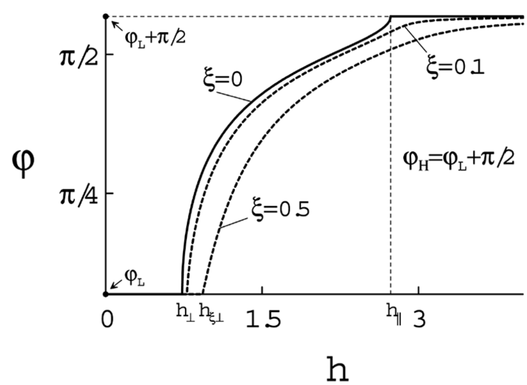


**FIGURE 5** The director angle $\varphi$ as a function of the magnetic field strength $h$ for the coupling energy $\sigma = 1$, the reactive parameter $\lambda = 1.1$, and for different values of the shear velocity gradient $\xi$.

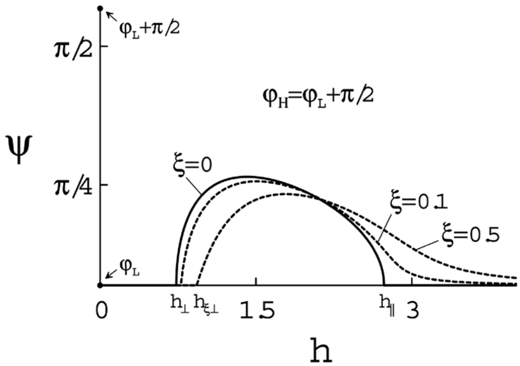


**FIGURE 6** The magnetization angle $\psi$ as a function of the magnetic field strength $h$ for the coupling energy $\sigma = 1$, the reactive parameter $\lambda = 1.1$, and for different values of the shear velocity gradient $\xi$.

Moreover, the increasing of $\xi$ "smoothes" more and more the threshold transition from the angular phase to the planar one. The maximum value of the tilt angle of the magnetization from the direction of field is decreased with the growth of $\xi$ at the given coupling energy $\sigma$.

## CONCLUSION

We have considered the steady state shear flow of unbounded ferronematic in the uniform magnetic field. Stationary solutions for the planar director and magnetization fields have been obtained as functions of the magnetic field strength, the coupling energy, and the velocity gradient of the shear flow. We have derived analytical expressions for the critical magnetic field strength, which determine the transitions between the orientational phases in weak shear flow, and asymptotic behavior for the director and magnetization in strong magnetic fields.

Without shear flow, in a ferronematic subjected to the uniform magnetic field the homeotropic, angular, and planar orientational phases can be observed at different magnetic field strength [2]. Our analysis shows that imposing of shear flow can leads to the shift of the field thresholds or to a "smoothing" of the transitions in a ferronematic.

In this article we have considered one of the possible mutual orientations of the magnetic field and direction of shear flow. The other configurations will be presented elsewhere.

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